How to Incorporate
Volatility and Risk In Electricity Price Forecasting

A Case Study Using an MMOPF Model[1]

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Introduction

Prior to deregulation and FERC Order 888, forecasts of electricity prices were mainly used for ratemaking and analysis of qualified facilities, and were therefore primarily focused on predicting the average and marginal costs of electricity. These forecasts were generally produced using single-commodity production-costing models. In the emerging competitive environment, investors, buyers and traders not only need insight into future electricity prices, but also assessment of the risks of buying and selling electricity at the forecast prices. The choice of a forecast, especially a forecast upon which a risk management strategy depends, must consider how completely the forecast represents and predicts risk.

Competition Requires a New Approach

In a competitive electricity market, the spot or day-ahead prices are not determined purely on a cost basis. Rather, they are based on the market participants' rational competitive behavior, and

their objective of maximizing income from all available markets, including the ancillary service and emission allowance markets. The traditional production-costing models do not represent the multi-commodity electricity market, ignore transmission constraints and neglect volatility; these models are therefore unsuitable for the emerging competitive

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electricity markets in today's environment. It has become apparent that in a market like California, accurate forecasting requires a multi-commodity, multi-area model capable of volatility simulation [1].

Risk Assessment is a Necessary Component of Forecasting

In addition to price forecasts, what kinds of information are necessary for managing risk? The call and put option values at the forecast price are valuable sources of information, which represent the buyers' and sellers' risks associated with a particular forecast. The key to option valuation and risk management is volatility, which refers to the swiftness with which a price changes, and often signifies a transition from one price regime to another. The electricity markets are known for high short-term volatility relative to that observed in the markets for more traditional commodities.

Why Traditional Methods are Inappropriate

One method of options valuation, based on the Black-Scholes or Black's models, relies on historical projections of the day-to-day volatility. While this method, based on Black's model in particular, has been successfully used for option valuation in many commodity markets, the application of Black's model to electricity prices as if electricity were a standard commodity does not provide predictions about volatility commensurate with the actual price risk. Some of the shortcomings of multiple-factor volatility analysis using historical projections are cited below.

➢ The electric system is subject to the confluence of unusually high demand, unexpected generator outages and transmission de-ratings.

➢ Instantaneous demand and supply imbalances subject the system to unusual stress; this can only be captured by hourly volatility, not by day-to-day volatility.

> ➢ Ancillary services (A/S), emission allowances (EA) and other products interact with energy prices and cannot be treated as isolated entities. Black's model is not suited for cross-market analysis, especially in California, New York, New England and Ontario, where such markets exist.
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> ➢ Past conditions are unlikely to be repeated in any consistent manner useful in forecasting.

We will provide empirical evidence of how the results of volatility analysis using historical time series of prices can be misleading, with the California Power Exchange (PX) as the focus of study.

We believe that by far the superior way to obtain accurate measures of electricity price risks over any period of time is by simulating the volatility of the fundamental drivers causing the electricity price swings with a Multi-commodity Multi-area Optimal Power Flow Model (MMOPF). Felak [2], writing about ideal software for traders, emphasizes the need for such models and asserts that "*chronological, security-constrained analysis of least-cost commitment and dispatch, with full AC optimal power-flow representation*" alone can provide "*location-specific spot prices* (italics added)."

The application of Black's model to electricity prices as if electricity were a standard commodity does not provide predictions about volatility commensurate with the actual price risk.

We will use a forecast of the 1998 and 1999 California PX prices, conducted for the California Energy Commission by LCG [3] in 1996, followed by a forecast of year 2000 PX prices and its volatility to make our point. The purpose of the 1996 forecast was to evaluate restructuring proposals for the California market prior to the enactment of California restructuring legislation. Readers may also be interested in a recent paper by Earle et al. [4] on the California experience, covering 1998 and 1999. Our overriding concern in this paper is the analysis and use of volatility, and how to incorporate risk through appropriate modeling of the underlying process.

This will also permit comparison of both the original and new methods' results with the actual market outcomes of 1998 and 1999. In time, the year 2000 forecast results can be compared with the actual prices of the entire current year. An analysis of the option values associated with the volatility is provided as part of this discussion.

# Role of Volatility in Electricity Prices and Risk Assessment

In most commodity markets, the price effects of production or supply-chain problems are dampened by surplus storage. By contrast, most electricity systems lack storage for all practical purposes. The electricity market therefore experiences pronounced short-term volatility due to the need for continuous balancing of demand and supply. Volatility in the electricity market is rooted in hourly, daily and seasonal uncertainty associated with fundamental market drivers and the physics of generation and delivery of electricity. A sudden heat wave can strain the ability of even backup generator capacity to meet elevated demand in a timely manner. The generators are subject to unexpected outages and changing emission constraints, while transmission lines may experience congestion, creating electrical imbalances.

When the actual value of any driver departs from what is used in a simulation, electricity prices can deviate significantly from the forecast. A point forecast based purely on the most likely, or expected, values of the drivers therefore gives only the most probable outcome for each hour. Such a forecast represents one sample path out of myriad potential sample paths.

The variability of the underlying drivers and the physical characteristics of the electricity market are the primary reasons for short-term volatility unknown in conventional commodity markets. Since the values of many fundamental drivers are highly uncertain over the long-term, an approach to electricity price forecasts based on single-point estimates of drivers is neither sufficient for determining market participants' day-to-day strategy nor suitable for asset valuation [5]. Thus, a more comprehensive forecast must capture the consequences of random and atypical fluctuations of fundamental market drivers and must be based on an accurate representation of the electrical system.

# Volatility Measures and Recent California Experience

In order to look at the contribution of market drivers to volatility, we will first examine the California PX prices in its first year of operation, April 1998 through March 1999, displayed in Figure 1. We use several volatility

Figure 1 - PX Hourly Prices
April 1998 - March 1999
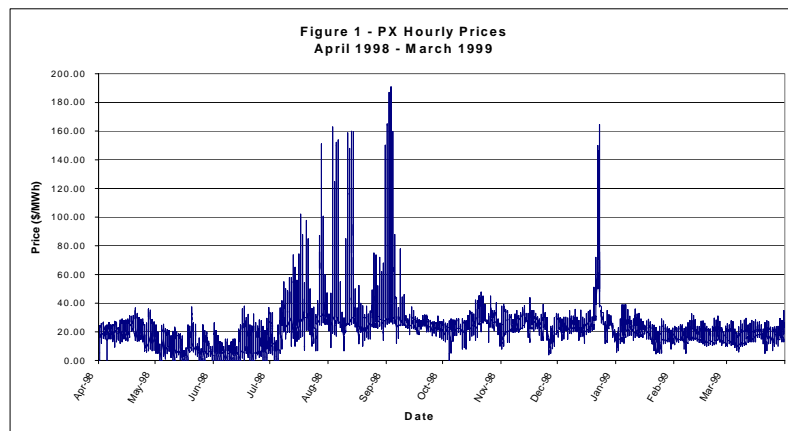
measures to understand what measures are useful in forecasting the randomness in electricity prices. The standard deviation is measured from the hourly prices within a day using the conventionally defined statistical formula. We define hourly log volatility[3] as the standard deviation of log of the ratio of prices during the same hour on successive days and daily log volatility[4] as log of the ratio of the average prices simulated by the model on consecutive days.

We will also use rolling volatility, which is calculated over fourteen days using the standard deviation of the log-ratio of daily average prices.

The standard deviation and the 14-day rolling volatility for April 1998 through March 1999 are shown

Figure 2 - Daily Standard Deviation and 14-Day Rolling Volatility
PX Prices April 1998 - March 1999

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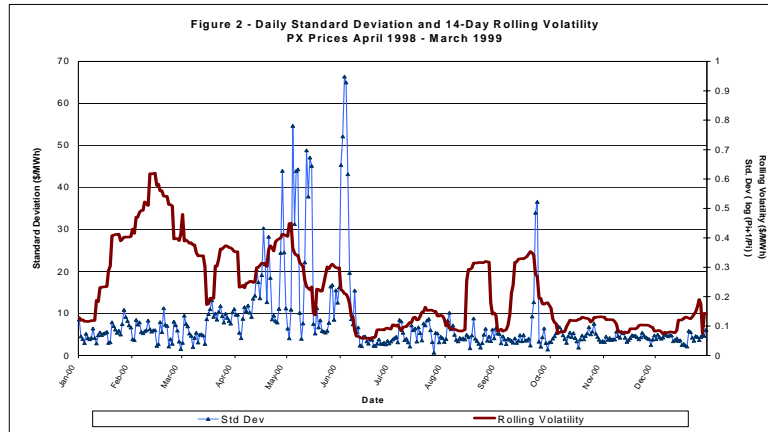
[3] The measure provided is the standard deviation of the log-ratios of the sample. The change $C(i,j,k)$ in price $P(i,j,k)$ on the ith day at the jth hour for the Monte Carlo sample k is given as

$$C(i,j,k) = \log\left(\frac{P(i+1, j, k)}{P(i, j, k)} \right)$$

Hourly volatility is the standard deviation of $C(i,j,k)$ for all the hours H (peak hours for on-peak and 24 hours for overall), and all the iterations N, and is equal to

$$\sqrt{ \left[\frac{\sum_{k=1}^{N} \sum_{j=1}^{H} C(i,j,k)^2}{N*H} - \left(\frac{\sum_{k=1}^{N} \sum_{j=1}^{H} C(i,j,k)}{N*H} \right)^2 \right] }$$

[4] In this formula for daily volatility, C is the log of next day's average price (for peak, off-peak or overall) over today's average price. The index for the day is i, j = 0 for off-peak prices, 1 for peak prices, and 2 for the entire day's hourly prices; k represents the sample.

$$C(i, j,k) = \log\left(\frac{AvgP(i+1, j, k)}{AvgP(i, j, k)} \right)$$

Daily volatility is the standard deviation of C for all iterations N and is equal to

$$\sqrt{ \left[\frac{\sum_{k=1}^{N} C(i,j,k)^2}{N} - \left(\frac{\sum_{k=1}^{N} C(i,j,k)}{N} \right)^2 \right] }$$ `

in Figure 2. The rolling volatility was high in the months May through early September 1998, although May and June had some of the lowest prices of the year and July and August prices were the highest. Since prices were low between March through June, the magnitude of rolling volatility was exaggerated by taking ratios of successive prices. The rolling volatility showed the impact of unusually high deviations dampened over time.

The standard deviation generally followed the PX price profile and was highest in the months of July, August and September, and low during April through June. In July through September, recurring fluctuations in load caused expensive generators to be taken on- and off-line frequently. This contributed to high price variations, and consequently, high volatility in both measures. The volatility measures were lower in the remaining months, up through March 1999.

These results indicate that rolling volatility does not discriminate between high and low prices. For example, if the price increases from $0.50 to $10 during off-peak time and from $80 to $1600 during on-peak hours, the log-ratio treats the increases as having contributed the same volatility. It appears that the standard deviation is an intuitively appealing measure of the variability in price. For instance, readers can translate one standard deviation above the mean as including prices that will be exceeded only 16% of the time.

1996 PX Study for CEC

LCG Consulting's proprietary structural model UPLAN[5] was used to forecast California electricity prices and to simulate the participants' behavior in the energy and ancillary markets. UPLAN is a Multi-commodity, Multi-area Optimal Power Flow (MMOPF) model with the ability to simulate volatility using Monte Carlo simulation. The model integrates the market participants and their rationally competitive bidding behavior, generation assets, the transmission network and its flow restrictions across interfaces. Each price driver is represented either in the bidding and scheduling sub-model, or in the real-time OPF model. The program not only incorporates the energy and ancillary service markets but also the interaction of energy prices with ancillary

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[5] UPLAN's Market Simulation Model is based on Rational Expected Equilibrium Prices (REEP) in the presence of multiple forward markets. The determination of competitive equilibrium prices in the presence of multiple markets as a non-linear game between the suppliers, who maximize their profits, and buyers, who minimize their payments. UPLAN uses a Very Very Large Scale Linear Program (VVLSLP), which alternates between minimizing the buyers' payment and maximizing sellers' marginal revenue in successive iterations until an equilibrium prices is reached. The UPLAN Network Power Model uses an optimal power flow (OPF) algorithm to dispatch the resources cleared by the Market Simulation Model to determine the real-time imbalance prices, calculates the security-constrained load flows, manages congestion and calculates transmission costs. UPLAN is a true Multi-commodity, Multi-area OPF (MMOPF) model.

service prices, as well as the emissions allowance market. The result of this multi-commodity simulation is an internally consistent forecast of prices across markets. Thus, the MMOPF-type model's ability to represent the physical resources with specificity, and to achieve an hourly balance of demand and supply resources sets it apart from any other form of modeling and provides unequaled accuracy in forecasting prices. In addition, such a model can be used for consistent analysis of volatility and thus, account for the large, unforeseen discrepancies that often occur between the forward and spot prices of electricity across time.

As has been discussed, a certain amount of variation in the conditions surrounding any market is anticipated. Nonetheless, a forecast must rely on inputs that are essentially consistent with past and expected conditions in the real world. The 1996 forecast were aimed at determining the most likely prices, predicated on a set of expected values of major variables or drivers.

Input Variables in 1996 PX Forecast

The California study [3] was based on the data developed by the California Energy Commission, the California Public Utility Commission, and the utilities in California (CFM 10). The data were crosschecked with the 1995 Load and Resources Report and the Path Rating Catalogue prepared by the Western System Coordinating Council, or WSCC (DOE Form OE-411) [6]. The hydrological conditions were based on a normal water year, as projected by the WSCC. Prices of natural gas and other fuels were derived from the California Energy Commission's biennial fuel price projection [7]. The demand forecast was developed from data provided by each of the individual utilities within California, while the loads outside California were based on WSCC projections. All of these fundamental drivers were treated as single point variables on a monthly basis and no provisions were made about any variability of these fundamental drivers.

1998-1999 PX Actuals' Impact on Prices

The input assumptions for the price forecast for 1998 turned out to be remarkably true in many regards, except for the hydrological conditions and the forecasted summer loads. This caused the actual PX prices for 1998 to follow a different trajectory than the forecast made for (CEC). The 1998 spring and winter hydro runoff turned out to be above average, with the spring water conditions increasing the availability of hydro energy by over 30%. There was a dip in the PX prices during this period, below what was forecasted. The 1998 summer peaks were also

higher than what was assumed in the study. The PX prices were above the forecast prices for these months, as higher-priced units participated to a greater extent than was forecasted (see Figure 3). These price differences were the direct result of substantial deviations from the expected behavior of two of the main drivers of price, loads and hydro availability.

Incorporating Volatility in Revised 1996Forecast

To illustrate the impact of the deviations in value of drivers, and to test the validity of the model's basic assumptions, a backcast, or revised forecast, was prepared for 1998-1999 by changing the summer load and water runoff in accordance with their reported actual values. Thus, hydro availability was increased during May and June and loads were increased during July and August.

Figure 3 - Daily Average PX Prices and UPLAN Forecast
April 1998 - March 1999
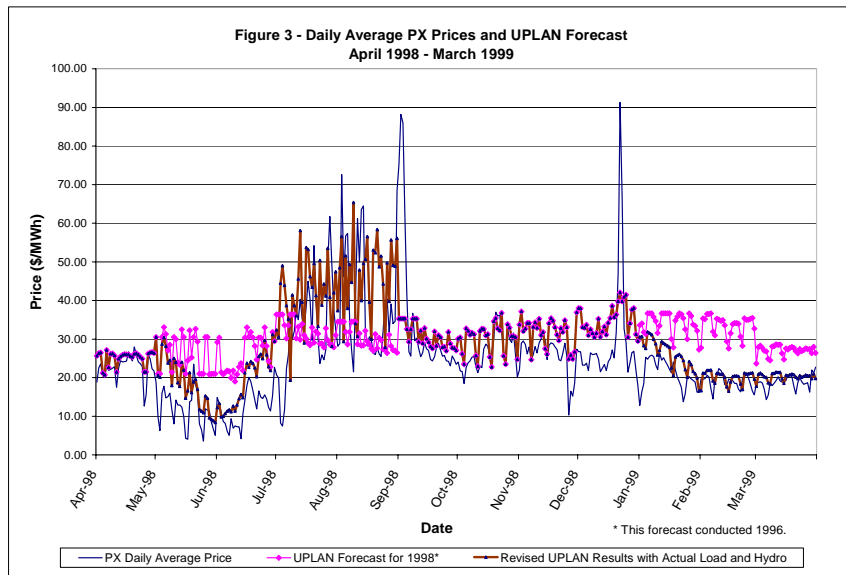
Looking at the results, the backcast prices move into correspondence with the actual prices. As shown in Figure 3, the effect of changing those few driver inputs was a new forecast noticeably closer to the actual prices. Thus, this simulation largely captured the dynamic effect of drivers on market behavior, validated the assumptions of the model and illustrated the capability of the program to replicate the market operations.

High summer loads and better-than-average hydro conditions offer but a few examples of the conditions with the potential to upset price expectations. In another year, outages might have occurred at a higher rate, temperate weather might have reduced loads below normal or hydrological conditions might have been adverse. Nonetheless, the different results of the two forecasts indicate the ability of a structural model to capture driver-price interaction. More importantly, the comparison underscores the importance of analyzing drivers' behavior for their impacts on the volatility of the price. This will lead to a more accurate analysis of price patterns within the limits of predictability.

Using Historical Data to the Best Advantage

With the 1998-1999 PX experience in mind, it is clear that a normal year with respect to every driver cannot be anticipated with any confidence. The question therefore remains, how can one use historical behavior of drivers in a forecast to the best advantage? A comprehensive forecast requires explicit representation of the variability of input drivers and their contribution to the probability distribution of prices and volatility. For the same reasons that point estimates of drivers are not appropriate, a more comprehensive form of forecast should include the probability distribution of prices. One can then use such distributions of the price to derive options values to hedge the risks associated with the forecast prices.

Volatility Analysis and Monte Carlo Simulation

One of the purposes of Monte Carlo simulation is to ascertain the systematic effect of market drivers upon price variability. When a structural model such as was used in this study generates a large number of Monte Carlo simulations of the system, it captures physical or instantaneous volatility as well as the temporal variation in price levels. We compare the volatility

> *A comprehensive forecast requires explicit representation of the variability of input drivers and their contribution to the probability distribution of prices and volatility.*

of the year 2000 forecast with the actual volatility of the PX prices from April 1998 through the latter part of 1999. The success of the model in capturing driver-price causality can be judged on the relative magnitude of the volatility observed throughout the year.

Forecast of PX Prices and Risk Assessment for 2000

For the year 2000 forecast and volatility analysis, a series of 100 Monte Carlo simulations were performed to determine the distribution of PX prices. Preparations for the volatility analysis involved the

Figure 4 - Monthly Average Prices and Ranges Forecast from UPLAN Forecast for Year 2000
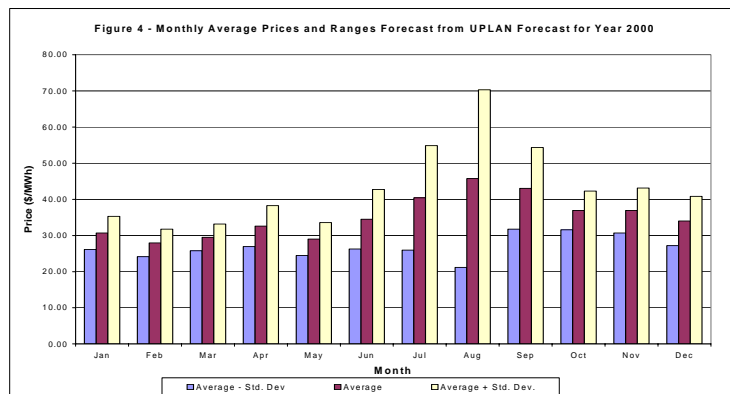
determination of probability distributions of the input variables, using historical values of these variables. Samples were drawn from four input variables, namely Loads, Hydro availability, Fuel Prices, and Transmission congestion. All relevant markets, including the PX energy, A/S and the real-time imbalance market, were modeled.

The simulations performed by the model provided daily distributions of future PX prices. Using these distributions, options values were calculated daily. Figure 4 presents the average monthly PX price forecast along with the prices at one standard deviation above and below the mean, all taken from the complete price distribution. Although the simulations were done for every hour of the year 2000, only monthly averages, and the prices at plus and minus one sigma are presented.

Figure 5 - Standard Deviation and Hourly Volatility Forecast for Year 2000
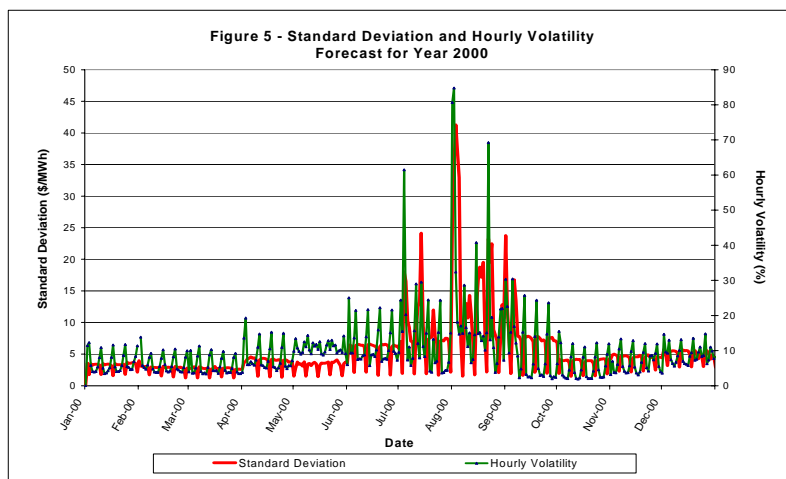
In Figure 5, the standard deviation and the hourly log volatility of the forecast are displayed. Note that two different scales were used for the ordinate axes of this graph. The volatility shown is the standard deviation over a day of the log-ratio of prices during the same hour on consecutive days.

The standard deviation of prices is greatest in the months of July and August, which also experience the highest monthly demand. A regular pattern of dips in the standard deviation throughout the year denotes the weekends, when load is normally lower and prices remain low relative to what is seen on weekdays. On the same days, during the changeover, the hourly log volatility displays spikes.

Comparison of Historical Volatility in the PX Market and in the 2000 Forecast
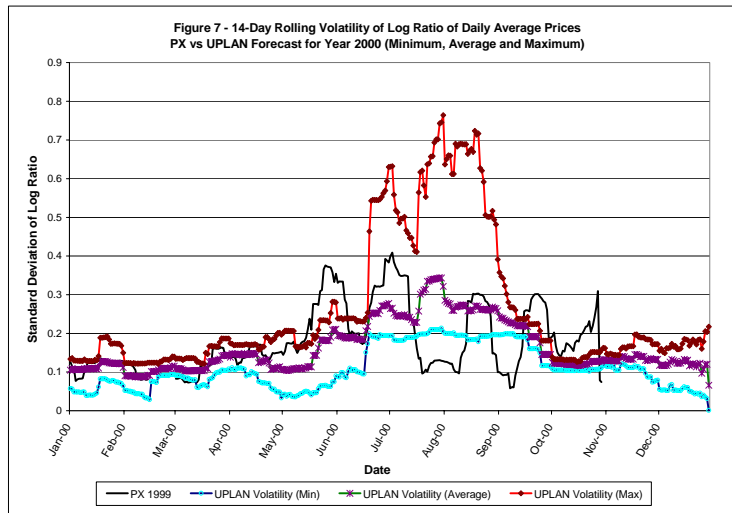
In Figure 6, the standard deviation of the hourly PX prices from 1998 and 1999 and from the UPLAN volatility output is presented. The year 2000 forecast's standard deviation was derived from all the 100 hourly simulations representing 100x24 samples per day, whereas the PX daily standard deviation was calculated from the 24 hourly prices on the given day. Therefore, the standard deviation of UPLAN is statistically more stable than that of the PX results.

The PX standard deviation in 1999 was not quite as persistently high as was the case in the same months of 1998. On the whole, however, the magnitude of the daily standard deviation was not very different between the two years' prices. A general seasonal pattern is clearly implied by all three sets of standard deviations in Figure 6. The forecast

Figure 6 - Standard Deviation (Daily)
PX and UPLAN 2000 Forecast
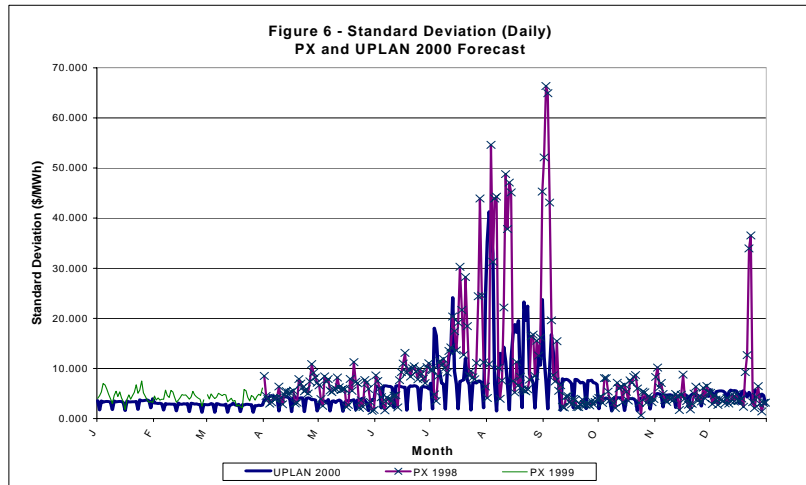
standard deviation is high in July and August, the peak-demand months of the year. During summer, the increased demand, coupled with larger short-term changes in weather, plays a significant role in the increase of the daily standard deviation. Unforeseen outages of generators or transmission de-ratings have larger impacts on volatility than do changes in fuel prices or water conditions. Also note that UPLAN's standard deviation follows the same annual pattern as the actual standard deviation except for May and June, due to low energy prices in the PX. If the block forward market recently introduced in June 1999 by the PX exhibits more liquidity, then we expect to see lower volatility in the summer months of year 2000.

In Figure 7, we present the 14-day rolling volatility, averaged over all simulation samples. Rolling volatility is presented on a daily basis for the 14-day rolling volatility averaged over all samples, as well as for the minimum and maximum 14-day rolling volatility observed over all the samples for each day. We

Figure 7 - 14-Day Rolling Volatility of Log Ratio of Daily Average Prices
**PX vs UPLAN Forecast for Year 2000 (Minimum, Average and Maximum)**

conclude that the simulated average volatility is consistent with the rolling volatility of 1999 PX prices. We will now turn to the evaluation of risk implied in the volatility of the forecast for the year 2000.
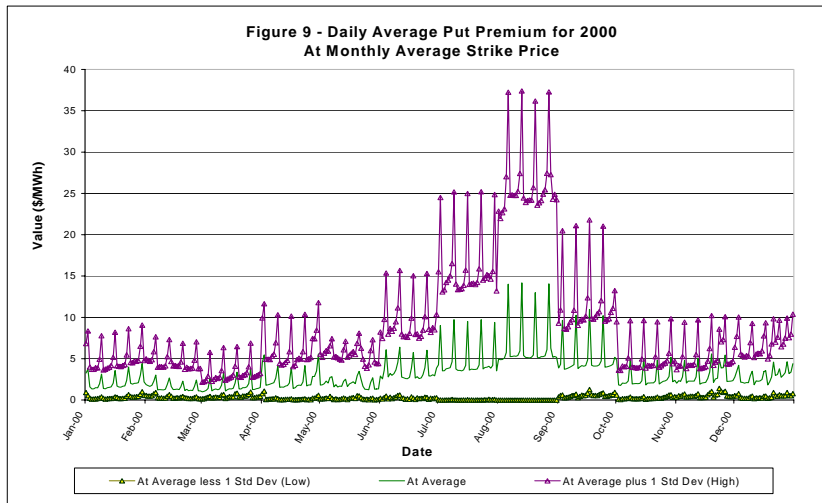
Risk and Options Valuation

   The key to options pricing for a commodity is the estimation of the spread between the forward price and the spot price. As mentioned, one of the well-known methods for options pricing of standard commodities is Black's model. It has been applied of late to evaluate options for electricity transactions as well. Black's model requires one to impute the volatility of the price from the historical data using some variant of Brownian motion with drift. Based on the computed volatility, options prices are deduced.

   Now, we will look into the financial risk associated with the deviation of actual prices from a particular forecast. One way to quantify this risk is to determine the expected premium a buyer would pay to make sure that he can conduct his trade at the average, or

Figure 8 - Daily Average Call Premium for 2000 At Monthly Average Strike Price
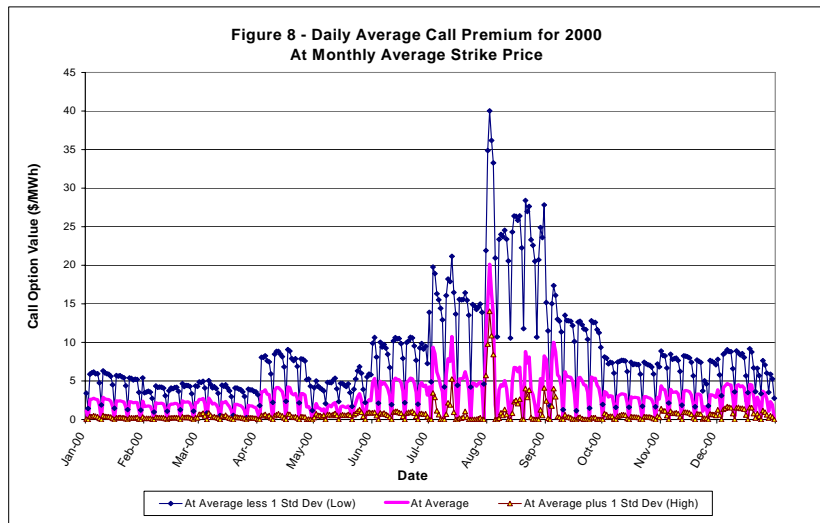
forecast price. This is exactly the call premium, while the corresponding risk premium for a

Figure 9 - Daily Average Put Premium for 2000 At Monthly Average Strike Price

seller wishing to ensure a sale at the average (strike) price is the value of the put at the strike price.   The values of both call and put options presented in Figures 8 and 9 are calculated directly from prices derived from the Monte Carlo simulations.[6] The

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[6] The value of the call option obtained from the hourly price distribution is the difference between the average of the set of prices above the mean and the mean price of the entire distribution. The value of the put option is the difference between the average of the prices less than the mean and the mean of the entire distribution.

results are given for three strike prices, corresponding to the average forecast, the average price minus one standard deviation ("low"), and the average plus one standard deviation ("high") respectively. Note that we can also calculate the option premiums at any strike price from the Monte Carlo samples.

As seen from Figure 8 and Figure 9, the call option's value drops on weekends with the narrowing of the standard deviation in the daily average price. By contrast, put option values peak on the weekend as the seller's risk increases.

Correlation between Volatility and Option Premium

Given that the risks conveyed by the instantaneous and temporal volatility are the primary determinants for an option's value, what measure of volatility is most suited for the option price calculation? We have tested the correlation between daily options values at monthly average

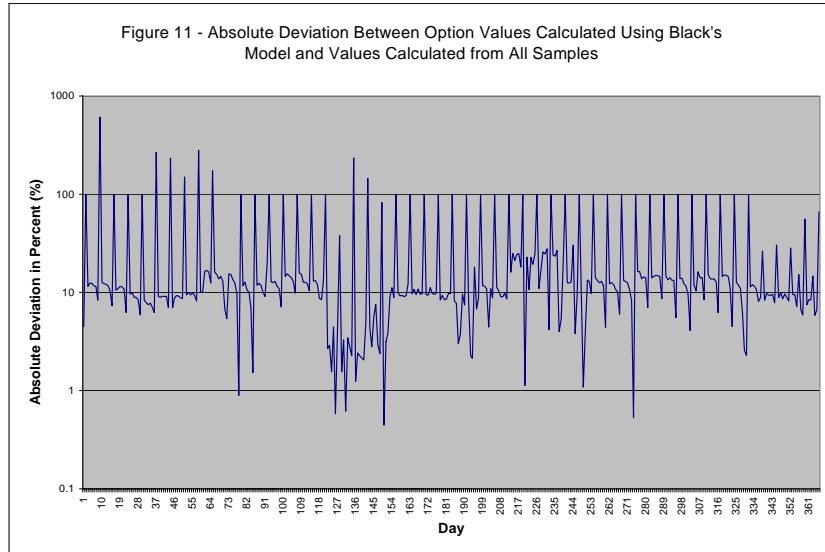Figure 10 - Correlation of Volatility Measurements with Call Option's Monthly Mean Value
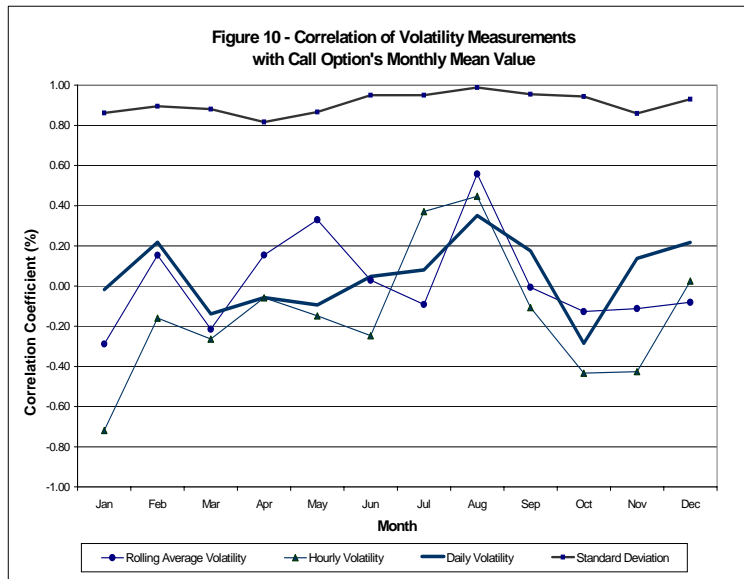
prices for the year 2000 with each of the following measures: rolling daily volatility, standard deviation, hourly volatility and daily volatility. The standard deviation differs from all the other measures, in that it measures price variability within a day while the others measure the variability between consecutive days. The results are displayed in Figure 10. The standard deviation emerges as the measure most strongly correlated with the options valuations, with a correlation averaging more than 80%. As seen in Figure 10, the average correlation between the options value and daily volatility is negligible and oscillates between negative and positive values. This is both a weak and inconsistent correlation, making daily volatility measured across successive days unsuitable for option valuation. Since the standard deviation measures the instantaneous structural volatility of the prices due to the interaction of the major fundamental drivers, it is a more desirable measure for electricity prices swings. Any model, which depends on historical data and derives volatility based on day-to-day changes from one available sample of the electricity prices is unable to explain most of the price volatility.

Comparing Option Premium and Estimating Error in Traditional Methods

We also looked into a hypothetical case in which we assumed that all the 100 simulation runs were available for Black-Scholes-type analysis. For this ideal case, the actual values of the options could be used as benchmarks against which to measure the accuracy of Black's model. The values of call option were calculated directly, using all the sample prices simulated by the model for the entire year. Corresponding call option values were also calculated using Black's formula and the volatility computed from the simulated prices. Normally this calculation

Figure 11 - Absolute Deviation Between Option Values Calculated Using Black's Model and Values Calculated from All Samples

is done using one historical time series; here, we were able to use all of the 100 samples to derive the volatility and hence this analysis will consistently provide a measure of error less than that inherent in any time-series analysis.

The percentage error between the option values obtained from using the sample distribution and the Black's formula were calculated and are displayed in Figure 11. In the graph, only the absolute difference is plotted in logarithmic scale due to very high differences. The error in Black's formula varies from a low of –100 percent to a high of +600 percent. This result clearly demonstrates that the lognormal assumption in Black's model is not appropriate for the electricity market and that a Multi-commodity, Multi-area OPF Model (MMOPF) is clearly a superior choice in evaluating electricity price risks.

Conclusion

Our analysis of the deregulated electricity market leads us to conclude that there is a strong need for internally consistent, accurate forecasting of prices and risks across all markets. The traditional multi-area models lack the sophistication necessary to satisfy the needs of the competitive market. At best, the forecasts that they may provide across product markets will

represent independent, piece-meal solutions to price prediction. In terms of risk assessment, traditional commodity option valuation using historical prices is unsuitable for the competitive electricity market and use of such methods more often than not will give misleading results. This is simply because predictions of the prices themselves and their volatility are intrinsically connected. A structural MMOPF-type model that performs Monte Carlo simulation to take into account all major drivers, including participants' bidding behavior, can provide reliable prices and realistic option values. These models can provide a firm basis for developing hedging strategies under uncertainty, an advantage not offered by outmoded regulatory-era models or conventional time-series price analysis.

References

1. Deb, R. K. 1999. Forecasting Energy and Ancillary Service Prices and Asset Evaluation. LCG Consulting.

2. Felak, R.P. 1995. Implementing Transmission Access: Getting the Genie Out of the Bottle (neck). *Public Utilities Fortnightly* 133 (May 1): 22-25.

3. LCG Consulting (Los Altos, Calif.). Modeling Competitive Energy Market in California: Analysis of Restructuring, prepared for California Energy Commission, Oct. 3, 1996. Principal investigator Rajat K. Deb, co-investigators Richard S. Albert and Lie-Long Hsue. http://www.energy.ca.gov/restructuring/papers/LCG_report.html.

4. Earle, R. L., Hanser, P. Q., Johnson, W. C., and J. D. Reitzes. 1999. Lessons from the First Year of Competition in the California Electricity Markets. The Electricity Journal 12 (October): 57-76.

5. Deb, R. K. 2000. Rethinking Asset Valuation in a Competitive Environment. *Public Utilities Fortnightly* 138 (February 1): 48 -55.

6. Western System Coordinating Council. April 1, 1995. *Coordinated Bulk Power Supply Program 1994-2004.* Replies to the United States Department of Energy Form OE-411 which requests information from the Regional Reliability Councils on Coordinated Regional Bulk Power Supply Programs.

7. California Energy Commission. 1995. Natural Gas Market Outlook. P300-95-017A.